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Two-dimensional magnetopolarons with squeezed Landau states

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Abstract. An attempt has been made to calculate the ground-state and first-excited-state energies of two-dimensional large magnetopolarons by using recently introduced squeezed Landau states. These states are obtained through the application of the two-mode coupled squeezed operator to the coherent states, and then a variational method is used to eliminate squeezing parameters. In the framework of this approach we obtain the ground- and first-excited-state energies of two-dimensional magnetopolarons.

1. Introduction

The coherent and squeezed states which play an important role in quantum optics [1] have now been found in condensed-matter physics. These states are interesting since, considering a quantum system, it is possible to make the uncertainties equal and minimal in the coherent states, and furthermore in the case of squeezed states uncertainty in one variance can be compressed at the expense of its complementary variance for two non-commuting operators, while keeping their product at the minimum value. These states are found to be particularly useful in obtaining lower quantum noise than the zero-point fluctuations of the coherent states of boson systems. This has been achieved for photons [1] and squeezed chaotic states [2], and its experimental realization for phonons has been proposed in recent works [3, 4]. As to the theoretical investigation, the squeezed states have been successfully applied to the calculation of various ground-state energies, and give more stable results than conventional considerations [5–7].

Another interesting problem in condensed-matter physics is the study of a particle in an external magnetic field, via which many physical concepts can be tested. In the presence of the external magnetic field, the energy of an electron splits into Landau levels. Starting from the first Landau level, i.e. the ground state, it is possible to form coherent states of an electron [8, 9]. Recently these coherent Landau states have been used to construct squeezed states, which show the same compression in uncertainties as arises in the case of quantum optics [10].

An electron moving slowly in a polar crystal polarizes the lattice, and this polarization in turn acts back on the electron to form a quasiparticle, called a polaron. Although the polaron concept is a well studied subject, an electron moving in a uniform magnetic field, which is known as the magnetopolaron problem, is still of current interest. In particular, the study of two-dimensional (2D) magnetopolarons, which is motivated by pioneering work of Das Sarma [11], Larsen [12], and Peeters and Devreese [13], has received increasing

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theoretical and experimental attention in recent years, because of the 2D electronic systems in semiconductor heterostructures [14, 15]. Despite the existence of detailed calculations for the 2D magnetopolaron effects, there is no study which combines the magnetopolaron and squeezed states together in an investigation.

In the present work, our aim is to reconsider 2D magnetopolarons in view of the recently introduced new squeezed Landau states [10]. In section 2 we introduce 2D magnetopolarons with some quantities of interest, such as relevant operators in the presence of a magnetic field. After giving a summary of coherent and squeezed Landau states in section 3, we calculate the energy functional by using coherent phonons. In the last section we discuss our results and compare the approach introduced in the previous sections with different theories.

2. Magnetopolarons in 2D

In the presence of a homogeneous magnetic field directed normal to the plane $z = 0$, the Hamiltonian of a 2D electron–phonon system is described by the Fröhlich Hamiltonian choosing the vector potential giving rise to a magnetic field $\boldsymbol{B}$ in the symmetric gauge, i.e. $\boldsymbol{A} \equiv (B/2)(-y, x, 0)$:

$$H_{2D} = H_0 + H_P + H_{EP} \tag{1}$$

where

$$H_0 = \frac{\boldsymbol{p}_\perp^2}{2\mu} + \frac{1}{2}\mu\left(\frac{\omega_c}{2}\right)^2 \boldsymbol{r}_\perp^2 - \frac{1}{2}\omega_c L_z \tag{2}$$

$$H_P = \sum_{\boldsymbol{k}_\perp} \hbar\omega_0 a_{\boldsymbol{k}_\perp}^\dagger a_{\boldsymbol{k}_\perp} \tag{3}$$

$$H_{EP} = \sum_{\boldsymbol{k}_\perp} \left[V_{\boldsymbol{k}_\perp} a_{\boldsymbol{k}_\perp} \exp(\mathrm{i}\boldsymbol{k}_\perp \cdot \boldsymbol{r}_\perp) + \mathrm{HC}\right]. \tag{4}$$

Here $\boldsymbol{p}_\perp \equiv (p_x, p_y)$ and $\boldsymbol{r}_\perp \equiv (x, y)$ are the momentum and the position of an electron confined to the plane $z = 0$. $L_z = xp_y - yp_x$ is the z-component of the orbital angular momentum operator and $\omega_c = eB/\mu$ is the cyclotron resonance frequency. $a_{\boldsymbol{k}_\perp}^\dagger (a_{\boldsymbol{k}_\perp})$ is the phonon creation (annihilation) operator with wave vector $\boldsymbol{k}_\perp \equiv (k_x, k_y)$. In (4), $V_{\boldsymbol{k}_\perp}$ is the 2D electron–phonon interaction amplitude, and is given by [16]

$$\left|V_{k_\perp}\right|^2 = \alpha \frac{2\pi}{V k_\perp} (\hbar\omega_0)^2 \left(\frac{\hbar}{2\mu\omega_0}\right)^{1/2} \tag{5}$$

where V is the surface area of the system, and α and ω_0 are respectively the electron–phonon coupling constant and the LO-phonon frequency, assumed to be dispersionless.

For the electronic parts of the Hamiltonian (1), the new dimensionless variables $(z, \bar{z})$, $(p_z, p_{\bar{z}})$ and the momentum-like operators $(\pi_z, \pi_{\bar{z}})$, $(\omega_{\bar{z}}, \omega_z)$ can be introduced in order to work in the energy basis of a 2D quantum oscillator [10]:

$$\pi_z = p_z + \frac{1}{2}\mathrm{i}\bar{z} \qquad \pi_{\bar{z}} = p_{\bar{z}} - \frac{1}{2}\mathrm{i}z \tag{6}$$

and

$$\omega_{\bar{z}} = p_{\bar{z}} + \frac{1}{2}\mathrm{i}z \qquad \omega_z = p_z - \frac{1}{2}\mathrm{i}\bar{z} \tag{7}$$

with

$$z = (\mu\omega_c/2\hbar)^{1/2}(x + \mathrm{i}y) \qquad p_z = (2\mu\hbar\omega_c)^{-1/2}(p_x - \mathrm{i}p_y) \tag{8}$$

where the bar denotes the complex conjugate. If we use these types of two decoupled sets of annihilation and creation operators, as in (6) and (7), for the energy $(\pi_z, \pi_{\bar{z}})$ and for the momentum $(\omega_{\bar{z}}, \omega_z)$ which satisfy the following commutation rules:

$$\left[\pi_{z,\bar{z}}, \omega_{z,\bar{z}}\right] = 0 \qquad [\pi_{\bar{z}}, \pi_z] = [\omega_z, \omega_{\bar{z}}] = 1 \tag{9}$$

then H_0, L_z and H_{EP} become

$$H_0 = \left(\pi_z \pi_{\bar{z}} + \frac{1}{2}\right)\hbar\omega_c \qquad L_z = \hbar(\pi_z\pi_{\bar{z}} - \omega_{\bar{z}}\omega_z) \tag{10}$$

and

$$H_{EP} = \sum_{\boldsymbol{k}_\perp}(V_{\boldsymbol{k}_\perp} a_{\boldsymbol{k}_\perp} \mathcal{L}_\pi \mathcal{M}_\omega + \mathrm{HC}) \tag{11}$$

where $\mathcal{L}_\pi$ and $\mathcal{M}_\omega$ are the exponential operator forms of the plane-wave part of (4) which is $\exp(\mathrm{i}\boldsymbol{k}_\perp \cdot \boldsymbol{r}_\perp)$, and can be written as

$$\exp(\mathrm{i}\boldsymbol{k}_\perp \cdot \boldsymbol{r}_\perp) = \mathcal{L}_\pi \mathcal{M}_\omega$$

with

$$\begin{aligned} \mathcal{L}_\pi &= \exp(K\pi_z - \bar{K}\pi_{\bar{z}}) \\ \mathcal{M}_\omega &= \exp(\bar{K}\omega_{\bar{z}} - K\omega_z). \end{aligned} \tag{12}$$

Here x and y have been expressed in terms of two decoupled sets of annihilation and creation operators defined in (6) and (7), such as

$$\begin{aligned} x &= \mathrm{i}\gamma\,[(\pi_{\bar{z}} - \omega_{\bar{z}}) - (\pi_z - \omega_z)] \\ y &= \gamma\,[(\pi_{\bar{z}} - \omega_{\bar{z}}) + (\pi_z - \omega_z)] \end{aligned} \tag{13}$$

in which $K = \gamma(k_x + \mathrm{i}k_y)$, and $\gamma = (\hbar/2\mu\omega_c)^{1/2}$ is the dimension of length. Taking the normalized ground state as $\Phi_{00} = |00\rangle$ for a 2D quantum oscillator in the form

$$\Phi_{00}(z\bar{z}) = \pi^{-1/2}\exp(-z\bar{z}/2) \tag{14}$$

the eigenfunctions of $\pi_z\pi_{\bar{z}}$ and $\omega_{\bar{z}}\omega_z$ can be generated in the usual way, that is, by operating with π_z and $\omega_{\bar{z}}$ on (14) repeatedly. One thus obtains the state vectors which are normalized to unity:

$$|n_1 n_2\rangle = (n_1! n_2!)^{-1/2} \pi_z^{n_1} \omega_{\bar{z}}^{n_2} |00\rangle\,. \tag{15}$$

3. Coherent and squeezed Landau states

3.1. Coherent Landau states

It is necessary to review some properties of coherent Landau states (CLS) which are in fact the eigenstates of the annihilation operators $\pi_{\bar{z}}$ and ω_z with eigenvalues p and w as follows:

$$\pi_{\bar{z}}\,|pw\rangle = p\,|pw\rangle \tag{16}$$

$$\omega_z\,|pw\rangle = \bar{w}\,|pw\rangle\,. \tag{17}$$

The CLS $|pw\rangle$ can also be defined as the simultaneous eigenstates of two commuting non-Hermitian operators which are created by the unitary displacement operators

$$D(p) = \exp(p\,\pi_z - \bar{p}\,\pi_{\bar{z}}) \tag{18}$$

$$D(w) = \exp(\bar{w}\omega_{\bar{z}} - w\omega_z) \tag{19}$$

acting on the ground state $|00\rangle$ to yield

$$|pw\rangle = D(p)D(w)\,|00\rangle = \exp\left[-\frac{1}{2}(|p|^2 + |w|^2)\right]\sum_{n_1,n_2}\frac{p^{n_1}\bar{w}^{n_2}}{\sqrt{n_1!n_2!}}\,|n_1n_2\rangle\,. \tag{20}$$

These are constructed out of states $|n_1n_2\rangle$, where p, w run separately over the entire complex plane. The CLS are normalized since $D(p)$ and $D(w)$ are unitary, and form an important identity or resolution of unity:

$$\frac{1}{\pi^2}\int\int \mathrm{d}^2p\ \mathrm{d}^2w\ |pw\rangle\langle pw| = \sum_{n_1,n_2}|n_1n_2\rangle\langle n_1n_2| = I \tag{21}$$

where $\mathrm{d}^2p = \mathrm{d}\,\mathrm{Re}(p)\,\mathrm{d}\,\mathrm{Im}(p)$ is the differential element of the area in the p-complex plane and similarly d^2w is that in the w-complex plane. The CLS are not orthogonal since the scalar product of two different states which are eigenstates of non-Hermitian operators given by (16) and (17) satisfies the overlap relation

$$\langle p_1w_1|p_2w_2\rangle = \exp\bigg[-\frac{1}{2}\,|p_{01}-p_{02}|^2 - \frac{1}{2}\,|w_{01}-w_{02}|^2 + \mathrm{i}\,|p_{02}|\,|p_{01}|\sin(\Phi_{12}-\Phi_{11}) - \mathrm{i}\,|w_{02}|\,|w_{01}|\sin(\Phi_{22}-\Phi_{21})\bigg] \tag{22}$$

where $p_i = |p_{0i}|\exp(\mathrm{i}\Phi_{1i})$ and $w_i = |w_{0i}|\exp(\mathrm{i}\Phi_{2i})$ are used. In fact, they are overcomplete as a result of the non-orthogonality. Hence the CLS must contain subsets which are complete. It is possible to choose such a subset, as first pointed out by von Neumann for the simultaneous measurements of both the coordinate and momentum in phase space. In the phase space this set forms a lattice with a unit-cell area of h. In the case of coherent states, the von Neumann set is chosen as a square lattice with the unit-cell area of π. This set is still overcomplete by just one element, but removing one element brings back completeness. For Landau levels a more general subset of coherent states is introduced by Dana [17]. The CLS are also minimum uncertainty states (MUS), i.e. the uncertainty relations $(\Delta x)_{CLS}(\Delta p_x)_{CLS} = (\Delta y)_{CLS}(\Delta p_y)_{CLS} = 1/2$ are satisfied. Furthermore, the energy and the angular momentum distribution in the state $|pw\rangle$ is a Poisson distribution:

$$P_{n_1n_2} = |\langle pw|n_1n_2\rangle|^2 = \exp\left[-(|p|^2+|w|^2)\right]\frac{|p|^{2n_1}|w|^{2n_2}}{n_1!n_2!}. \tag{23}$$

In the CLS representation the expectation values for the H_0 and L_z given by (10) can be easily found, by using (18) and (19), as

$$E_p = \left(|p|^2 + \frac{1}{2}\right)\hbar\omega_c \qquad L_z = \hbar(|p|^2 - |w|^2) \tag{24}$$

where p and w take all of the values over the whole of the complex planes as pointed out before.

3.2. Squeezed Landau states

In order to obtain the squeezed Landau states (SLS), it is necessary to consider the coupled squeeze operator which is defined by [10]

$$D(q) = \exp\left(\frac{1}{2}q^2\pi_z\omega_{\bar{z}} - \frac{1}{2}\bar{q}^2\pi_{\bar{z}}\omega_z\right) \tag{25}$$

where $q^2 = r\exp(2\mathrm{i}\varphi)$. Thus, the SLS may be written as

$$|pw, q\rangle = D(q)D(p)D(w)|00\rangle \tag{26}$$

where the operators $D(p)$ and $D(w)$ displace and the operator $D(q)$ squeezes the ground state, i.e. $D(q)$ squeezes the CLS defined by (20). Using (25), one obtains the transformed forms of operators $\pi_{\bar{z}}$ and ω_z as

$$\pi_{\bar{z}q} = D^{-1}(q)\pi_{\bar{z}}D(q) = \pi_{\bar{z}}\cosh\left(\frac{1}{2}r\right) + \exp(2\mathrm{i}\varphi)\omega_{\bar{z}}\sinh\left(\frac{1}{2}r\right) \tag{27}$$

$$\omega_{zq} = D^{-1}(q)\omega_z D(q) = \omega_z\cosh\left(\frac{1}{2}r\right) + \exp(2\mathrm{i}\varphi)\pi_z\sinh\left(\frac{1}{2}r\right). \tag{28}$$

In order to obtain the uncertainties in (x, p_x) and (y, p_y), these are firstly expressed in terms of operators (6) and (7), as was done before for the calculation of uncertainties in associated coherent observables, and then the squeezed expectation values of the related operators can be obtained by means of (27) and (28). As a result of the calculation of these uncertainties, it can be easily seen that

$$(\Delta x)^2_{SLS}(\Delta p_x)^2_{SLS} = (\Delta y)^2_{SLS}(\Delta p_y)^2_{SLS} = \frac{1}{4}\left[\cosh^2(r) - \sinh^2(r)\cos^2(2\varphi)\right]. \tag{29}$$

In addition, these uncertainties are MUS for $\varphi = k\pi/2$, with integer k. This means that one squeezes either in (x, p_x) or in (y, p_y) as follows:

$$(\Delta x)^2_{SLS} = (\Delta p_y)^2_{SLS} = \frac{1}{2}\left[\cosh(r) - (-1)^k\sinh(r)\right]$$

$$(\Delta y)^2_{SLS} = (\Delta p_x)^2_{SLS} = \frac{1}{2}\left[\cosh(r) + (-1)^k\sinh(r)\right]$$

and by choosing k as even or odd respectively:

$$(\Delta x)^2_{SLS} = (\Delta p_y)^2_{SLS} = \frac{1}{2}\exp(\pm r)$$

$$(\Delta y)^2_{SLS} = (\Delta p_x)^2_{SLS} = \frac{1}{2}\exp(\mp r).$$

In order to find the squeezed expectation values for the energy and the angular momentum of the electronic parts of (1), one has to write the transformed forms of (27) and (28) in terms of (18) and (19), which simply correspond to displacements in $(\pi_z,\pi_{\bar{z}})$ and $(\omega_{\bar{z}},\omega_z)$ by the amounts $(\bar{p},p)$ and $(w,\bar{w})$. After forming H_0 and L_z with these new operators, their expectation values can be calculated from the ground state, and it is found that

$$E_{pw}(r,\varphi) = \left[\left(|p|^2 + \frac{1}{2}\right)\cosh^2(r/2) + \left(|w|^2 + \frac{1}{2}\right)\sinh^2(r/2)\right.$$
$$\left. + \ \frac{1}{2}(\bar{p}w\exp(2\mathrm{i}\varphi) + p\bar{w}\exp(-2\mathrm{i}\varphi))\sinh(r)\right]\hbar\omega_c \tag{30}$$

and

$$L_z = \hbar(|p|^2 - |w|^2). \tag{31}$$

Here it appears that L_z remains invariant under the squeezed transformation from its form in (24).

3.3. Coherent phonons

In (4), the electron–phonon interaction Hamiltonian is not in a diagonal form in phonon coordinates, but can be diagonalized by applying a unitary displacement operator which is also coherent in $a_{\boldsymbol{k}}$, and gives the phonon state as $|f\rangle = D(f)\,|0\rangle_{PH}$ with

$$D(f) = \exp\left\{\sum_{\boldsymbol{k}_\perp}\left[a^\dagger_{\boldsymbol{k}_\perp} f_{\boldsymbol{k}_\perp} - a_k f^*_{\boldsymbol{k}_\perp}\right]\right\}. \tag{32}$$

This transforms the phonon operators as

$$D^{-1}(f) a_{\boldsymbol{k}_\perp} D(f) = a_{\boldsymbol{k}_\perp} + f_{\boldsymbol{k}_\perp} \tag{33}$$

$$D^{-1}(f) a^\dagger_{\boldsymbol{k}_\perp} D(f) = a^\dagger_{\boldsymbol{k}_\perp} + f^*_{\boldsymbol{k}_\perp} \tag{34}$$

and it is known as the Lee–Low–Pines (LLP) transformation [18]. Here $f_{\boldsymbol{k}_\perp}$ will be used as a variational parameter to be determined by minimizing the total energy of the electron–phonon system. Then the state vector of an electron interacting with LO phonons in 2D can be taken as

$$|pw, q;\, f\rangle = D(f)|0\rangle_{PH} \otimes D(q)D(p)D(w)|00\rangle. \tag{35}$$

The transformed forms of the Hamiltonian of the free-phonon field plus the electron–phonon interaction have the form

$$\begin{aligned} D^{-1}(f)(H_P + H_{EP})D(f) &= \sum_{\boldsymbol{k}_\perp}\left[\hbar\omega_0\left|f_{\boldsymbol{k}_\perp}\right|^2 + (V_{\boldsymbol{k}_\perp} f_{\boldsymbol{k}_\perp}\mathcal{L}_\pi\mathcal{M}_\omega + \mathrm{HC})\right] \\ &\quad + \sum_{\boldsymbol{k}_\perp}\left\{\hbar\omega_0 a^\dagger_{\boldsymbol{k}_\perp} a_{\boldsymbol{k}_\perp} + \left[a_{\boldsymbol{k}_\perp}(f^*_{\boldsymbol{k}_\perp} + V_{\boldsymbol{k}_\perp}\mathcal{L}_\pi\mathcal{M}_\omega) + \mathrm{HC}\right]\right\} \end{aligned} \tag{36}$$

where the normal ordering of operators has been used. We can write down the total energy functional of a 2D polaron:

$$E_{2D}\left[pw, q;\, f\right] = \langle pw, q;\, f|\, H\, |pw, q;\, f\rangle$$

and then separately calculate the electronic part $E_{pw}(r,\varphi)$ and the part containing phonons and the interaction with phonons $E_\alpha(r,\varphi)$:

$$E_{2D}\left[pw, q;\, f\right] = E_{pw}(r,\varphi) + E_\alpha(r,\varphi). \tag{37}$$

In the last equation, $E_{pw}(r,\varphi)$ and $E_\alpha(r,\varphi)$ are given by

$$\begin{aligned} E_{pw}(r,\varphi) &= \left[\left(|p|^2 + \frac{1}{2}\right)\cosh^2(r/2) + \left(|w|^2 + \frac{1}{2}\right)\sinh^2(r/2)\right. \\ &\quad \left. + \frac{1}{2}(\bar{p}w\exp(2\mathrm{i}\varphi) + p\bar{w}\exp(-2\mathrm{i}\varphi))\sinh(r)\right]\hbar\omega_c \end{aligned} \tag{38}$$

$$\begin{aligned} E_\alpha(r,\varphi) &= \sum_{k_\perp}\left[\hbar\omega_0\left|f_{\boldsymbol{k}_\perp}\right|^2\right] \\ &\quad + \sum_{k_\perp}\left[V_{k_\perp} f_{\boldsymbol{k}_\perp}\langle 00|\,\bar{\mathcal{L}}_{\pi q}\bar{\mathcal{M}}_{\omega q}\,|00\rangle + V^*_{k_\perp} f^*_{\boldsymbol{k}_\perp}\langle 00|\,\bar{\mathcal{L}}^{-1}_{\pi q}\bar{\mathcal{M}}^{-1}_{\omega q}\,|00\rangle\right] \end{aligned} \tag{39}$$

In (39), $\bar{\mathcal{L}}_{\pi q}$ and $\bar{\mathcal{M}}_{\omega q}$ are respectively the transformed forms of the exponential operators (12) under the unitary transformations $D(q)$, $D(p)$ and $D(w)$ and are defined by

$$\begin{aligned} \bar{\mathcal{L}}_{\pi q}\bar{\mathcal{M}}_{\omega q} &= D^{-1}(p)D^{-1}(w)\mathcal{L}_{\pi q}\mathcal{M}_{\omega q}D(p)D(w) \\ &= D^{-1}(p)D^{-1}(w)\exp(K\pi_{zq} - \bar{K}\pi_{\bar{z}q})\exp(\bar{w}\omega_{\bar{z}q} - w\omega_{zq})D(p)D(w) \\ &= \exp\left\{-|K|^2\left[\cosh(r) - \sinh(r)\cos(2\varphi)\right]\right\} \\ &\quad \times \exp\left\{+(K\bar{p} + \bar{K}w)\left[\cosh(r/2) - \sinh(r/2)\exp(+2\mathrm{i}\varphi)\right]\right\} \\ &\quad \times \exp\left\{-(\bar{K}p + K\bar{w})\left[\cosh(r/2) - \sinh(r/2)\exp(-2\mathrm{i}\varphi)\right]\right\} \\ &\quad \times \exp(K_{r\varphi}\pi_z)\exp(-\bar{K}_{r\varphi}\pi_{\bar{z}})\exp(\bar{K}'_{r\varphi}\omega_{\bar{z}})\exp(-K'_{r\varphi}\omega_z) \end{aligned} \tag{40}$$

where

$$K_{r\varphi} = K\left[\cosh(r/2) - \sinh(r/2)\exp(2\mathrm{i}\varphi)\right]$$
$$K^{'}_{r\varphi} = \bar{K}\left[\cosh(r/2) - \sinh(r/2)\exp(2\mathrm{i}\varphi)\right]$$

and the operators are written in the normal ordered forms by using the Baker, Campbell and Hausdorff (BCH) formula.

By minimizing (37) with respect to $f_{\boldsymbol{k}_\perp}$, r and φ, we obtain respectively the relevant parameters in the following coupled equations:

$$f_{\boldsymbol{k}_\perp} = -\frac{V_{k_\perp}}{\hbar\omega_0}\langle 00|\,\bar{\mathcal{L}}_{\pi q}\bar{\mathcal{M}}_{\omega q}\,|00\rangle \tag{41}$$

$$\left[(|p|^2 + |w|^2 + 1)\sinh(r) + (\bar{p}w\exp(2\mathrm{i}\varphi) + p\bar{w}\exp(-2\mathrm{i}\varphi))\cosh(r)\right]\hbar\omega_c + \frac{4}{\hbar\omega_0}\sum_{\boldsymbol{k}_\perp}\left|V_{\boldsymbol{k}_\perp}\right|^2|K|^2 g'(r,\varphi)\exp(-2\,|K|^2 g(r,\varphi)) = 0 \tag{42}$$

$$\mathrm{i}(\bar{p}w\exp(2\mathrm{i}\varphi) - p\bar{w}\exp(-2\mathrm{i}\varphi))\sinh(r)\hbar\omega_c + \frac{4}{\hbar\omega_0}\sum_{\boldsymbol{k}_\perp}\left|V_{\boldsymbol{k}_\perp}\right|^2|K|^2\sinh(r)\sin(2\varphi)\exp(-2\,|K|^2 g(r,\varphi)) = 0 \tag{43}$$

where $g \equiv g(r,\varphi) = [\cosh(r) - \sinh(r)\cos(2\varphi)]$ and g' is the derivative of g with respect to r. The last two equations can be written in a simpler form by changing sums over $\boldsymbol{k}_\perp$ to integrals, and using the expression for $V_{k_\perp}$ given in (5) and K defined above. After integration these are expressed as

$$\left[(|p|^2 + |w|^2 + 1)\sinh(r) + (\bar{p}w\exp(2\mathrm{i}\varphi) + p\bar{w}\exp(-2\mathrm{i}\varphi))\cosh(r)\right]\bar{\Omega} + \alpha\bar{\Omega}^{1/2}\sqrt{\frac{\pi}{2^3}}g'(r,\varphi)g^{-3/2}(r,\varphi) = 0 \tag{44}$$

$$\mathrm{i}(\bar{p}w\exp(2\mathrm{i}\varphi) - p\bar{w}\exp(-2\mathrm{i}\varphi))\sinh(r)\bar{\Omega} + \alpha\bar{\Omega}^{1/2}\sqrt{\frac{\pi}{2^3}}\sinh(r)\sin(2\varphi)g^{-3/2}(r,\varphi) = 0 \tag{45}$$

where $\bar{\Omega} = \omega_c/\omega_0$.

Since the last two coupled equations are complicated dependencies on α, $\bar{\Omega}$, r and φ, it seems impossible to solve them; however, one can obtain the following equations by setting $\alpha = 0$ as a first approximation. In (45), by taking $p = p_0\exp(\mathrm{i}\Phi_1)$ and $w = w_0\exp(\mathrm{i}\Phi_2)$ we obtain the squeezing angle that satisfies the MUS condition:

$$\varphi = \frac{1}{2}(\Phi_1 - \Phi_2) \tag{46}$$

and

$$\frac{1}{\cosh(r)} = \frac{\left[(p_0^2 + w_0^2 + 1)^2 - 4p_0^2w_0^2\right]^{1/2}}{(p_0^2 + w_0^2 + 1)} \tag{47}$$

or equivalently

$$2p_0w_0 = -(p_0^2 + w_0^2 + 1)\tanh(r). \tag{48}$$

Using (41) in (39) and rewriting (38) in the dimensionless form we find

$$\bar{E}_{pw}(r,\varphi) = \left[\left(|p|^2 + \frac{1}{2}\right)\cosh^2(r/2) + \left(|w|^2 + \frac{1}{2}\right)\sinh^2(r/2)\right.$$

$$+\frac{1}{2}(\bar{p}w\exp(2\mathrm{i}\varphi)+p\bar{w}\exp(-2\mathrm{i}\varphi))\sinh(r)\Big]\bar{\Omega} \tag{49}$$

$$\begin{aligned}\bar{E}_\alpha(r,\varphi) &= -\frac{1}{(\hbar\omega_0)^2}\sum_{k_\perp}\left|V_{k_\perp}\right|^2\left|\langle 00|\,\bar{\mathcal{L}}_{\pi q}\bar{\mathcal{M}}_{\omega q}\,|00\rangle\right|^2 \\ &= -\frac{1}{(\hbar\omega_0)^2}\sum_{k_\perp}\left|V_{k_\perp}\right|^2\exp(-2|K|^2k_\perp^2 g(r,\varphi)) \\ &= -\alpha\frac{1}{2}\sqrt{\frac{\pi}{2}}\bar{\Omega}^{1/2}g^{-1/2}(r,\varphi)\end{aligned} \tag{50}$$

where $\bar{E}_{pw}(r,\varphi)=E_{pw}(r,\varphi)/\hbar\omega_0$ and $\bar{E}_\alpha(r,\varphi)=E_\alpha(r,\varphi)/\hbar\omega_0$.

Substituting the expressions (46) and (47) into (49) and (50), the parts of the total energy can be found in terms of p, w:

$$\bar{E}_{p_0w_0}=\left\{\frac{1}{2}(p_0^2-w_0^2)+\frac{1}{2}\left[(1+p_0^2+w_0^2)^2-4p_0^2w_0^2\right]^{1/2}\right\}\bar{\Omega} \tag{51}$$

and

$$\begin{aligned}\bar{E}_\alpha(p_0,w_0) &= -\alpha\frac{1}{2}\sqrt{\frac{\pi}{2}}\bar{\Omega}^{1/2}\frac{\left[(1+p_0^2+w_0^2)^2-4p_0^2w_0^2\right]^{1/4}}{(1+p_0^2+w_0^2)^{1/2}} \\ &\quad\times\left[1+\frac{2p_0w_0}{(1+p_0^2+w_0^2)}\cos(\Phi_1-\Phi_2)\right]^{-1/2}.\end{aligned} \tag{52}$$

It should be noted that (51) gives the Landau ground-state energy when we take $p_0=0$, that is, if the coherence property vanishes. In order to make a comparison with other work one may transform the continuous (p_0,w_0) to discrete values as discussed above in the completeness argument. Alternatively one can project the coherent states on the Fock space.

If we multiply both (51) and (52) by the related Poisson distribution (23) and integrate over p and w, we obtain the following equations:

$$\begin{aligned}\bar{E}_{n_1n_2}(\bar{\Omega}) &= \left\{\frac{1}{2}(n_1-n_2)+\frac{2}{n_1!n_2!}\int_0^\infty\int_0^\infty \mathrm{d}p_0\,\mathrm{d}w_0\;p_0^{2n_1+1}w_0^{2n_2+1}\exp\left[-(p_0^2+w_0^2)\right]\right. \\ &\quad\left.\times\left[(1+p_0^2+w_0^2)^2-4p_0^2w_0^2\right]^{1/2}\right\}\bar{\Omega}\end{aligned} \tag{53}$$

and

$$\begin{aligned}\bar{E}_{n_1n_2}(\alpha,\bar{\Omega}) &= -\alpha\frac{1}{2}\sqrt{\frac{\pi}{2}}\bar{\Omega}^{1/2}\frac{1}{\pi^2}\frac{1}{n_1!n_2!}\int_0^\infty\int_0^\infty \mathrm{d}p_0\,\mathrm{d}w_0\;p_0^{2n_1+1}w_0^{2n_2+1} \\ &\quad\times\exp\left[-(p_0^2+w_0^2)\right]\frac{\left[(1+p_0^2+w_0^2)^2-4p_0^2w_0^2\right]^{1/4}}{(1+p_0^2+w_0^2)^{1/2}} \\ &\quad\times\int_0^{2\pi}\int_0^{2\pi}\mathrm{d}\Phi_1\,\mathrm{d}\Phi_2\left[1+\frac{2p_0w_0}{(1+p_0^2+w_0^2)}\cos(\Phi_1-\Phi_2)\right]^{-1/2}.\end{aligned} \tag{54}$$

It should be mentioned that, on taking $n_1=n_2=0$ in (53), one obtains not the ground-state energy, but an energy level shifted down due to squeezing effects, which will be identified as the first-excited-state energy of magnetopolarons. In order to perform the integration over the angles in (53), it is useful to transform them by choosing the variables

Φ_1 and Φ_2 as

$$\begin{aligned}\Phi_1 &= (\Theta_1 + \Theta_2)/2 \\ \Phi_2 &= (\Theta_2 - \Theta_1)/2.\end{aligned} \tag{55}$$

Thus the integral takes the form

$$\begin{aligned} I(k) &= \int_0^{2\pi}\int_0^{2\pi} \mathrm{d}\Phi_1\, \mathrm{d}\Phi_2\, [1 + k^2\cos(\Phi_1 - \Phi_2)]^{-1/2} \\ &= 2\pi\int_0^{2\pi} \mathrm{d}\Theta_2\, [1 + k^2\cos(\Theta_2)]^{-1/2} \end{aligned} \tag{56}$$

where $k = 2p_0w_0/(1+p_0^2+w_0^2)$. The last equation can be expressed in terms of an elliptic function of the second kind:

$$I(k) = 2\pi\left\{2\sqrt{k^4}\left[\frac{E\left[2k^2/(k^2-1)\right]}{k^2\sqrt{1-k^2}} + \frac{E\left[2k^2/(k^2+1)\right]}{k^2\sqrt{1+k^2}}\right]\right\}. \tag{57}$$

Using this last integral, equations (53) and (54) can be calculated numerically by giving integer values to n_1 and n_2.

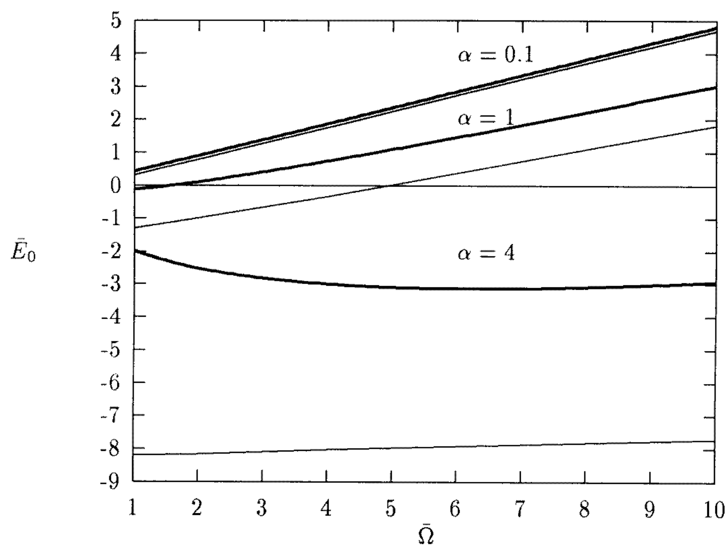

Figure 1. The ground-state energy of a 2D magnetopolaron in units of $\hbar\omega_0$ as a function of $\bar{\Omega} = \omega_c/\omega_0$. The fine curves show the numerical results of [23] and the bold curves are obtained from equation (58), for $\alpha = 0.1$, 1 and 4 respectively.

4. Results and discussion

In our approach we have used standard coherent states that arise from the ground state [19]. Let us first consider pure Landau states—that is, ignore electron–phonon interaction. The expectation value of H_0 is given by (24), in which there exists only $|p|^2$ of the coherent parameter. If the coherence vanishes we obtain the ground-state energy—that is, $\hbar\omega_c/2$. If we now integrate (24) after multiplying by P_{00} of (23), we obtain $(1 + 1/2)\hbar\omega_c$, i.e. the second Landau level. In fact, the other coherent states coincide with the normal Landau levels, as one unit shifted up. If we now consider the squeezed Landau energy given by (51), we notice that it gives the ground state upon taking $p_0 = 0$. This is an expected result, since one cannot shift the ground-state energy. If we now integrate (51) after multiplying by P_{00}, we obtain $1.230\,73\hbar\omega_c$, which is shifted down from the coherent Landau level $1.5\hbar\omega_c$. In the framework of our approach we will take this as the first excited state which we compare with other results.

Our approach makes use of the LLP method and the variational principle; it is therefore valid in weak- and intermediate-coupling regions and for cyclotron frequencies higher than ω_0, which are reached at high magnetic fields [20]. The ground-state energy of the 2D electron–phonon system which follows from (51) and (52) can be obtained by taking $p_0 = 0$ for any value of w_0 in a unique way as

$$\bar{E}_0 = \frac{1}{2}\bar{\Omega} - \frac{\alpha}{2}\sqrt{\frac{\pi}{2}}\bar{\Omega}^{1/2}. \tag{58}$$

This result agrees well with those from the path integral method obtained by Larsen [21] and the variational method [22] in high magnetic fields. Figure 1 shows the variation of this energy as a function of $\bar{\Omega}$ for various values of α. It should be mentioned that the lowest ground-state energy obtained so far is due to Wu *et al* [23], and their numerical results are also plotted in figure 1.

As far as the ground state is concerned, our approach agrees fairly well with the best available result only for low values of α—for example, when $\alpha \leqslant 1$. As α increases our result seems to be inadequate. This is due to the fact that the coherence and consequently the squeezing effects have to be removed in order to obtain the ground state.

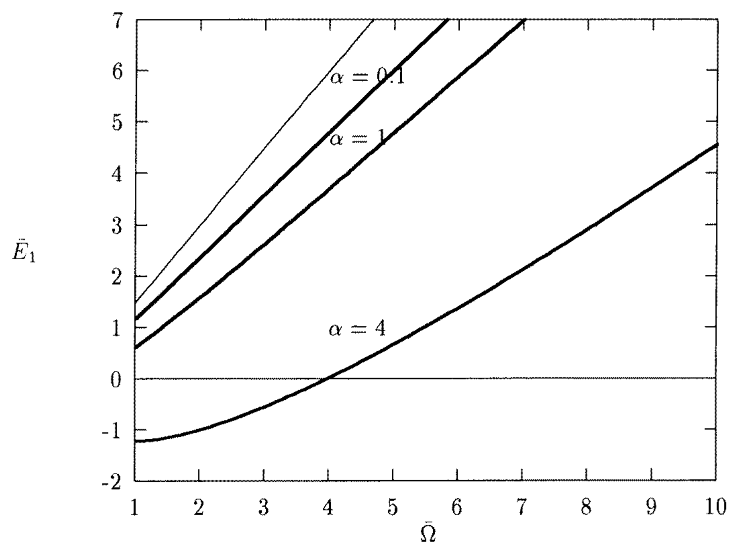

Figure 2. The first-excited-state energy of a 2D magnetopolaron in units of $\hbar\omega_0$ as a function of $\bar{\Omega} = \omega_c/\omega_0$. The fine curve shows the unperturbed second Landau level $n = 1$ and the bold curves are obtained from the equation (59), for $\alpha = 0.1$, 1 and 4 respectively.

The first-excited-state energy of the 2D electron–phonon system in our approach can be obtained from (53) and (54) by taking $n_1 = 0$ and $n_2 = 0$, as discussed above. The corresponding energy expression is

$$\bar{E}_1 = 1.230\,73\,\bar{\Omega} - 0.768\,164\,\alpha\sqrt{\frac{2}{\pi}}\,\bar{\Omega}^{1/2}. \tag{59}$$

Note that the second Landau state is lowered by the squeezing effects. This energy is plotted against $\bar{\Omega}$ for various values of α in figure 2. Although there is a similar calculation for 3D magnetopolarons within the same intermediate region [20], there is no work available in 2D to compare our result with. However, our result justifies the assumption that the polaron effects are considerably enhanced in 2D systems.

In this paper, we have presented an approach which makes use of coherent and squeezed Landau states. We have identified the ground and first excited states in the framework of this approach. As an application for these new states we have considered 2D magnetopolarons. We expect our model to be useful in similar problems involving Landau levels.

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